Design Reference: 70% Ethanol at Pc = 200 psi, r = 1.1

$$T = 100 \cdot \left| 4.448222 \cdot \frac{N}{lbf} \right|$$ *Desired thrust*

$g = 9.81$ [m/s^2] *Gravity*

$I_{sp} = 220.7$ [s] *Specific impulse from RPA*

$$\dot{m} = \frac{T}{I_{sp} \cdot g}$$ *Calculate propellant mass flow rate*

Constant gas properties

$k = 1.141$ *Specific heat ratio from RPA*

$R = 0.3727$ [kJ/kg-K] *Gas constant from RPA*

Chamber properties

$P_c = 1379$ [kPa] *Chamber pressure*

$T_c = 2873$ [K] *Combustion temperature from RPA*

$$\rho_c = \frac{P_c}{R \cdot T_c}$$ *Combustion chamber gas density*

Throat: M = 1

$$\frac{P_c}{P_t} = \left[1 + \left(\frac{k - 1}{2}\right) \cdot M_t^2\right]^{\left[\frac{k}{k - 1}\right]}$$ *Pressure at the throat*

$$\frac{T_c}{T_t} = 1 + \left[\frac{k - 1}{2}\right] \cdot M_t^2$$ *Temperature at the throat*

$$\rho_t = \frac{P_t}{R \cdot T_t}$$ *Density at the throat*

$$c_t = \sqrt{k \cdot R \cdot \left| 1000 \cdot \frac{J/kg\text{–}K}{kJ/kg\text{–}K} \right| \cdot T_t}$$ *Speed of sound at the throat*

$M_t = 1$

$V_t = c_t \cdot M_t$ *Velocity at the throat*

$\dot{m} = \rho_t \cdot V_t \cdot A_t$ *Calculate throat area from knowns in mass flow rate equation*

$$A_t = \pi \cdot \frac{D_t^2}{4}$$ *Calculate throat diameter from throat area*

Exit: P_e = 101.3 kPa

$P_e = 101.3$ [kPa] *Expand to atmospheric pressure*

$$\frac{P_c}{P_e} = \left[1 + \left(\frac{k - 1}{2}\right) \cdot M_e^{\,2}\right]^{\left[\frac{k}{k - 1}\right]}$$ *Calculate exit Mach number from chember/exit pressure ratio*

$$\frac{T_c}{T_e} = 1 + \left[\frac{k - 1}{2}\right] \cdot M_e^{\,2}$$ *Exit gas temperature*

$$\rho_e = \frac{P_e}{R \cdot T_e}$$ *Exit gas density*

$$c_e = \sqrt{k \cdot R \cdot \left|1000 \cdot \frac{\text{J/kg–K}}{\text{kJ/kg–K}}\right| \cdot T_e}$$ *Speed of sound at the exit*

$$V_e = c_e \cdot M_e$$ *Velocity at the exit*

$$\dot{m} = \rho_e \cdot V_e \cdot A_e$$ *Calculate exit area from mass flow rate*

$$A_e = \pi \cdot \frac{D_e^{\,2}}{4}$$ *Calculate exit diameter from exit area*

$$\text{ThroatDiameter} = D_t \cdot \left|39.37 \cdot \frac{\text{in}}{\text{m}}\right|$$

$$\text{ExitDiameter} = D_e \cdot \left|39.37 \cdot \frac{\text{in}}{\text{m}}\right|$$

Chamber dimensions

$$\varepsilon_c = 8$$ *Contraction ratio*

$$A_c = \varepsilon_c \cdot A_t$$ *Chamber cross sectional area*

$$A_c = \pi \cdot \frac{D_c^{\,2}}{4}$$ *Throat diameter*

$$\text{ChamberDiameter} = D_c \cdot \left|39.37 \cdot \frac{\text{in}}{\text{m}}\right|$$

$$\theta = 20$$ *Convergence half angle*

$$L_c = 5 \cdot \left|0.0254 \cdot \frac{\text{m}}{\text{in}}\right|$$ *Chamber length*

$$L_{conv} = \frac{\frac{D_c - D_t}{2}}{\mathbf{tan}\left[\theta\right]}$$ *Length of chamber converging section*

$$V_{conv} = \pi \cdot \frac{L_{conv}}{12} \cdot \left[D_c^{\,2} + D_t \cdot D_c + D_t^{\,2}\right]$$ *Volume of the converging part*

$$V_c = A_c \cdot L_c + V_{conv}$$ *Chamber volume*

$$L_{star} = \frac{V_c}{A_t} \cdot \left|39.37 \cdot \frac{\text{in}}{\text{m}}\right|$$

Size propellant tanks based on propellant densities and required mass flow

$mixtureratio = 1.1$

$\dot{m}_f = \dfrac{\dot{m}}{1 + mixtureratio}$ *Fuel mass flow rate from mixture ratio and total mass flow*

$\dot{m}_o = \dot{m} - \dot{m}_f$ *Oxidizer mass flow rate*

$time = 30$ [s] *Burn duration*

$m_f = \dot{m}_f \cdot time$ *Required fuel mass*

$m_o = \dot{m}_o \cdot time$ *Required oxidizer mass*

$\rho_O = 1141$ [kg/m^3] *Oxidizer density*

$\rho_F = 786$ [kg/m^3] *Fuel density*

$V_o = \dfrac{m_o}{\rho_O}$ *Oxidizer volume*

$V_f = \dfrac{m_f}{\rho_F}$ *Fuel volume*

$OxVolume = V_o \cdot \left| 264.2 \cdot \dfrac{gal}{m^3} \right|$

$FuVolume = V_f \cdot \left| 264.2 \cdot \dfrac{gal}{m^3} \right|$

Required pressurant mass and volume

$T_g = 298$ [K] *Nitrogen temperature*

$R_g = \dfrac{8.314 \text{ [kJ/kmol-K]}}{\textbf{MolarMass}[N2]} \cdot \left| 1000 \cdot \dfrac{J}{kJ} \right|$ *Gas constant for N2*

$k_g = \textbf{k}_s [N2, T = T_g]$ *Isentropic exponent for nitrogen*

$P_p = 400 \cdot \left| 6895 \cdot \dfrac{Pa}{psi} \right|$ *Propellant tank pressure*

$P_g = 3000 \cdot \left| 6895 \cdot \dfrac{Pa}{psi} \right|$ *Pressurant storage pressure*

$m_{go} = P_p \cdot V_o \cdot \dfrac{k_g}{R_g \cdot T_g \cdot \left[1 - \dfrac{P_p}{P_g} \right]}$ *Mass of N2 required to pressurize the oxidizer tank*

$m_{gf} = P_p \cdot V_f \cdot \dfrac{k_g}{R_g \cdot T_g \cdot \left[1 - \dfrac{P_p}{P_g} \right]}$ *Mass of N2 required to pressurize the fuel tank*

$V_g = [m_{go} + m_{gf}] \cdot R_g \cdot \frac{T_g}{P_g}$ *Volume of N2 pressurant tank*

$N2Volume = V_g \cdot \left| 264.2 \cdot \frac{gal}{m^3} \right|$ *Volume of N2 pressurant tank in gallons*

Size injector orifices. Plan A - unlike doublets. Subject to change

$C_{do} = 0.7$

$C_{df} = 0.7$

$\delta P = P_p - P_c \cdot \left| 1000 \cdot \frac{Pa}{kPa} \right|$ *Pressure drop across injector*

$\dot{m}_f = C_{df} \cdot A_{if} \cdot \sqrt{2 \cdot \rho_F \cdot \delta P}$ *Solve for required fuel orifice area from fuel mass flow rate*

$\dot{m}_o = C_{do} \cdot A_{io} \cdot \sqrt{2 \cdot \rho_o \cdot \delta P}$ *Solve for required oxidizie orifice area from ox mass flow rate*

$N_{pair} = 6$ *N_{pairs} of unlike doublets*

$A_{if} = N_{pair} \cdot \pi \cdot \frac{D_{if}^2}{4}$

$A_{io} = N_{pair} \cdot \pi \cdot \frac{D_{io}^2}{4}$

$OxDiam = D_{io} \cdot \left| 39.37 \cdot \frac{in}{m} \right|$ *Ox orifice diameter in inches*

$FuDiam = D_{if} \cdot \left| 39.37 \cdot \frac{in}{m} \right|$ *Fuel orifice diameter in inches*

SOLUTION

Unit Settings: SI K kPa kJ mass deg

A_c = 0.001937 [m²]	A_e = 0.0007194 [m²]
A_{if} = 0.000003002 [m²]	A_{io} = 0.000002741 [m²]
A_t = 0.0002421 [m²]	ChamberDiameter = 1.955 [in]
C_{df} = 0.7	C_{do} = 0.7
c_e = 940.6 [m/s]	c_t = 1068 [m/s]
δP = 1.379E+06 [Pa]	D_c = 0.04966 [m]
D_e = 0.03027 [m]	D_{if} = 0.0007981 [m]
D_{io} = 0.0007626 [m]	D_t = 0.01756 [m]
ε_c = 8	ExitDiameter = 1.192 [in]
FuDiam = 0.03142 [in]	FuVolume = 0.9865 [gal]
g = 9.81 [m/s²]	I_{sp} = 220.7 [s]
k = 1.141	k_g = 1.401
L_c = 0.127 [m]	L_{conv} = 0.0441 [m]
L_{star} = 46.85 [in]	mixtureratio = 1.1
$\dot{m}$ = 0.2055 [kg/s]	$\dot{m}_f$ = 0.09784 [kg/s]
$\dot{m}_o$ = 0.1076 [kg/s]	M_e = 2.324

m_f = 2.935 [kg]
m_{go} = 0.1426 [kg]
M_t = 1
N_{pair} = 6
OxVolume = 0.7475 [gal]
P_e = 101.3 [kPa]
P_p = 2.758E+06 [Pa]
R = 0.3727 [kJ/kg-K]
ρ_e = 0.1306 [kg/m^3]
ρ_o = 1141 [kg/m^3]
R_g = 296.8 [J/kg-K]
θ = 20 [deg]
time = 30 [s]
T_e = 2081 [K]
T_t = 2684 [K]
V_{conv} = 0.00004209 [m^3]
V_f = 0.003734 [m^3]
V_o = 0.00283 [m^3]
m_{gf} = 0.1882 [kg]
m_o = 3.229 [kg]
N2Volume = 0.3736 [gal]
OxDiam = 0.03002 [in]
P_c = 1379 [kPa]
P_g = 2.068E+07 [Pa]
P_t = 794.6 [kPa]
ρ_c = 1.288 [kg/m^3]
ρ_F = 786 [kg/m^3]
ρ_t = 0.7944 [kg/m^3]
T = 444.8 [N]
ThroatDiameter = 0.6912 [in]
T_c = 2873 [K]
T_g = 298 [K]
V_c = 0.0002881 [m^3]
V_e = 2186 [m/s]
V_g = 0.001414 [m^3]
V_t = 1068 [m/s]

No unit problems were detected.